Closed loop control of QETU for Ground State Preparation

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Agenda

- Intro: Ground state preparation via QETU
- Simulating noise in QETU
- A closed loop control scheme
- Numerical validation

Intro: Ground state preparation via QETU

- Many quantum algorithms for preparing the ground state of a Hamiltonian (e.g. VQE, QPE, Adiabatic evolution)
  - Each with existing tradeoffs

- **Quantum Eigenvalue Transformation (QETU)** offers a framework with low query complexity and a singular ancilla

$$H_{\text{TFIM}} = -\sum_{j=0}^{n-2} Z_j Z_{j+1} - g \sum_{j=0}^{n-1} X_j$$

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QETU Advantage across the rest of the field

TABLE II. A comparison of the performance of quantum algorithms for ground-state preparation in terms of the query complexity, the query depth, the number of ancilla qubits, and the level of multiqubit control (abbreviated as "MQC" in the table) operations needed. γ is the overlap between the initial guess $|\phi_0\rangle$ and the ground state, Δ is a lower bound of the spectral gap, and $1-\epsilon$ is the target fidelity. "HE" denotes the Hamiltonian-evolution model (assuming no ancilla qubits) and "BE" the block-encoding model. Here, we assume that an upper bound of the ground-state energy is known (μ in Theorem 6). The algorithm in Ref. [37] (GTC19 in the table) requires precise knowledge of the ground-state energy. We assume that Ref. [1] uses m ancilla qubits and that the high-level MQC operation is due to the block encoding of H.

	Query depth	Query complexity	Number of ancilla qubits	Need MQC?	Input model
This work (Theorem 6)	$\widetilde{\mathcal{O}}(\Delta^{-1})$	$\widetilde{\mathcal{O}}(\Delta^{-1}\gamma^{-2})$	$\mathcal{O}(1)$	No	HE
This work (Theorem 11)	$\widetilde{\mathcal{O}}(\Delta^{-1}\gamma^{-1})$	$\widetilde{\mathcal{O}}(\Delta^{-1}\gamma^{-1})$	$\mathcal{O}(1)$	Low	HE
QPE (high confidence) [41–43]	$\widetilde{\mathcal{O}}(\Delta^{-1})$	$\widetilde{\mathcal{O}}(\Delta^{-1}\gamma^{-2})$	$\mathcal{O}(\mathrm{poly}\log(\Delta^{-1}\gamma^{-1}\epsilon^{-1}))$	High	HE
QPE (semiclassical) [44,45]	$\widetilde{\mathcal{O}}(\Delta^{-1}\gamma^{-2})$	$\widetilde{\mathcal{O}}(\Delta^{-1}\gamma^{-4})$	$\mathcal{O}(1)$	No	HE
GTC19 (Theorem 1) [37]	$\widetilde{\mathcal{O}}(\Delta^{-1}\gamma^{-1})$	$\widetilde{\mathcal{O}}(\Delta^{-1}\gamma^{-1})$	$\mathcal{O}(\log(\Delta^{-1})+\log\log(\epsilon^{-1}))$	High	HE
LT20 [1]	$\widetilde{\mathcal{O}}(\Delta^{-1}\gamma^{-1})$	$\widetilde{\mathcal{O}}(\Delta^{-1}\gamma^{-1})$	m	High	BE

Important to note, there now exist other methods that are more efficient than QETU. but QETU is still a necessary algorithm for downstream tasks

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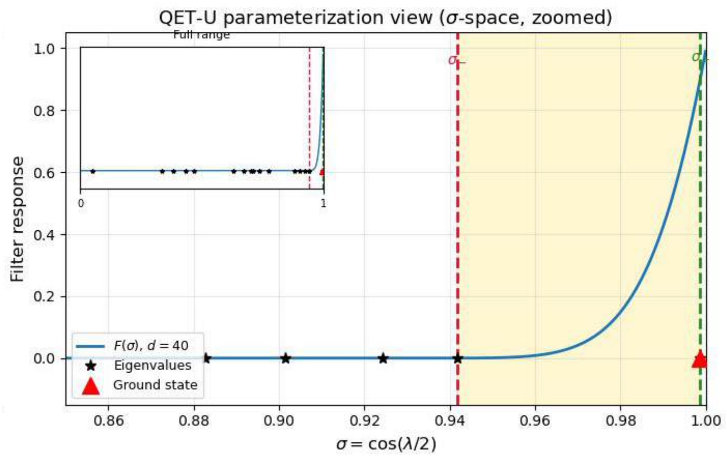
QETU filters for ground state

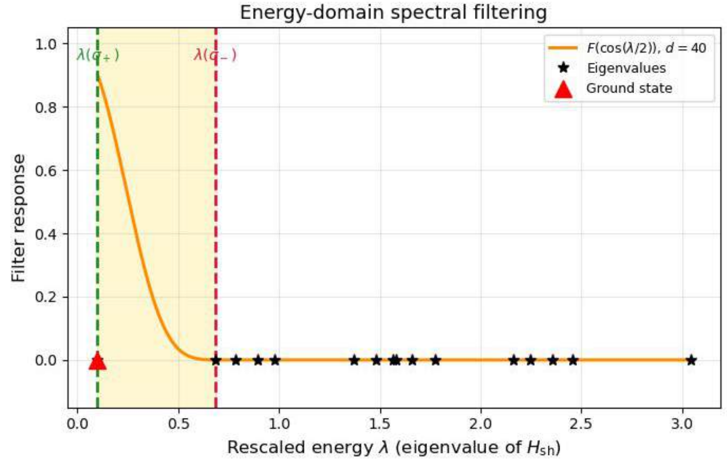
- **Goal:** apply a function F(H) to $|\psi_{init}\rangle$ such that $F(E_0) \approx 1$ and $F(E_k) \approx 0$ for all excited states

$$F(H)\,|\psi_{\rm init}\rangle = c_0 F(E_0)\,|\psi_0\rangle + \sum_{k\geq 1} c_k F(E_k)\,|\psi_k\rangle$$

- **QET-U realizes this as a polynomial** $F(\sigma)$ in $\sigma=\cos(\lambda/2)$, where $\lambda \in [0,\pi]$ is the rescaled energy

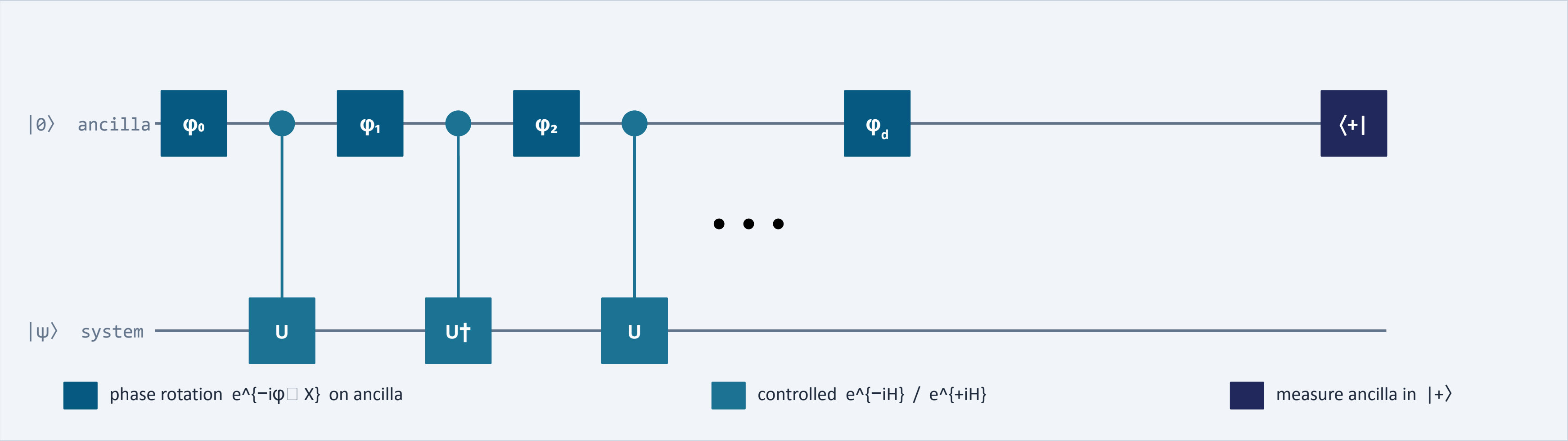
- Sharper filter → better projection





The QETU circuit in detail

The phase angles $\{\phi_j\}$ are computed classically so that the circuit implements the target polynomial F(cos(H/2))



On success (ancilla measured in |+⟩): system register is left in F(cos(H/2))|ψ⟩ the designed filter is applied to each eigenstate.
On failure (|−⟩): a different polynomial was applied; discard and rerun.

$$U = e^{-iH_{\mathrm{tfim}}}$$

$$R_x(\phi_j) = e^{-i\phi_j X/2}$$

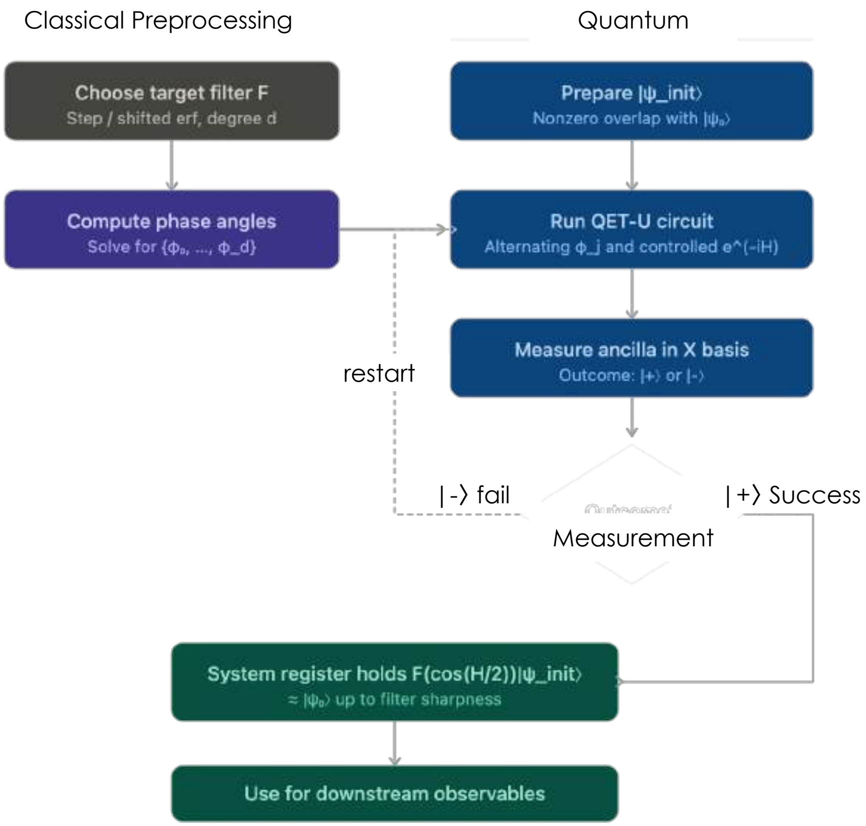
Quantum Signal Processing

- QSP is used to perform polynomial transformation on matrices
- Foundational framework of many quantum algorithms
- Achieves this by applying a sequence of specific rotations, parameterized by **phase factors**

$$U(x,\Phi) = \begin{pmatrix} p(x) & r(x) \\ r^*(x) & p^*(x) \end{pmatrix} = e^{i\phi_0 Z} e^{i\arccos(x)X} e^{i\phi_1 Z} e^{i\arccos(x)X} \cdots e^{i\phi_{d-1} Z} e^{i\arccos(x)iX} e^{i\phi_d Z}$$

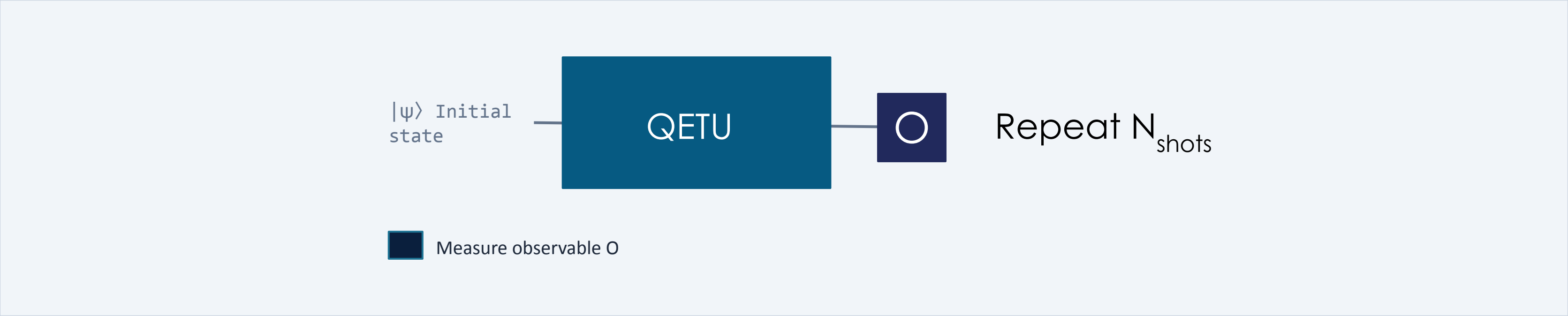
$$U_x \equiv \begin{bmatrix} x & * \\ * & * \end{bmatrix} \Rightarrow U_{f(x)} \equiv \begin{bmatrix} f(x) & * \\ * & * \end{bmatrix}.$$

QETU as a whole



From ground state to observables

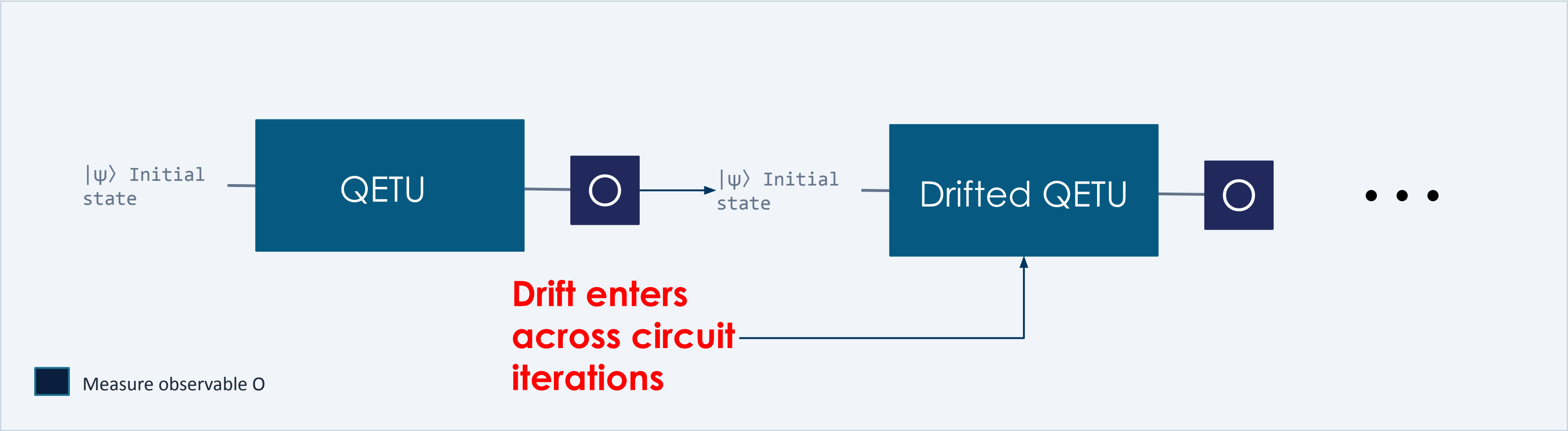
- We now take the prepared ground state and measure on it. Typical observables are $\langle \psi_0 | O | \psi_0 \rangle$
  - Energy ($O = H$), correlation functions, order parameters
- Run QET-U many times → collect successes → measure O on each → average gives $\langle O \rangle \approx \langle \psi_0 | O | \psi_0 \rangle$ up to filter approximation error.
- We study **energy density** $\langle H \rangle / N$, thermodynamic-limit observable



Introducing noise to QETU

Hardware Defects

- QETU makes assumptions about exact implementation of gates
- Real hardware suffers from calibration drift over large timescale
    - Timescales required for measuring observables on the ground state



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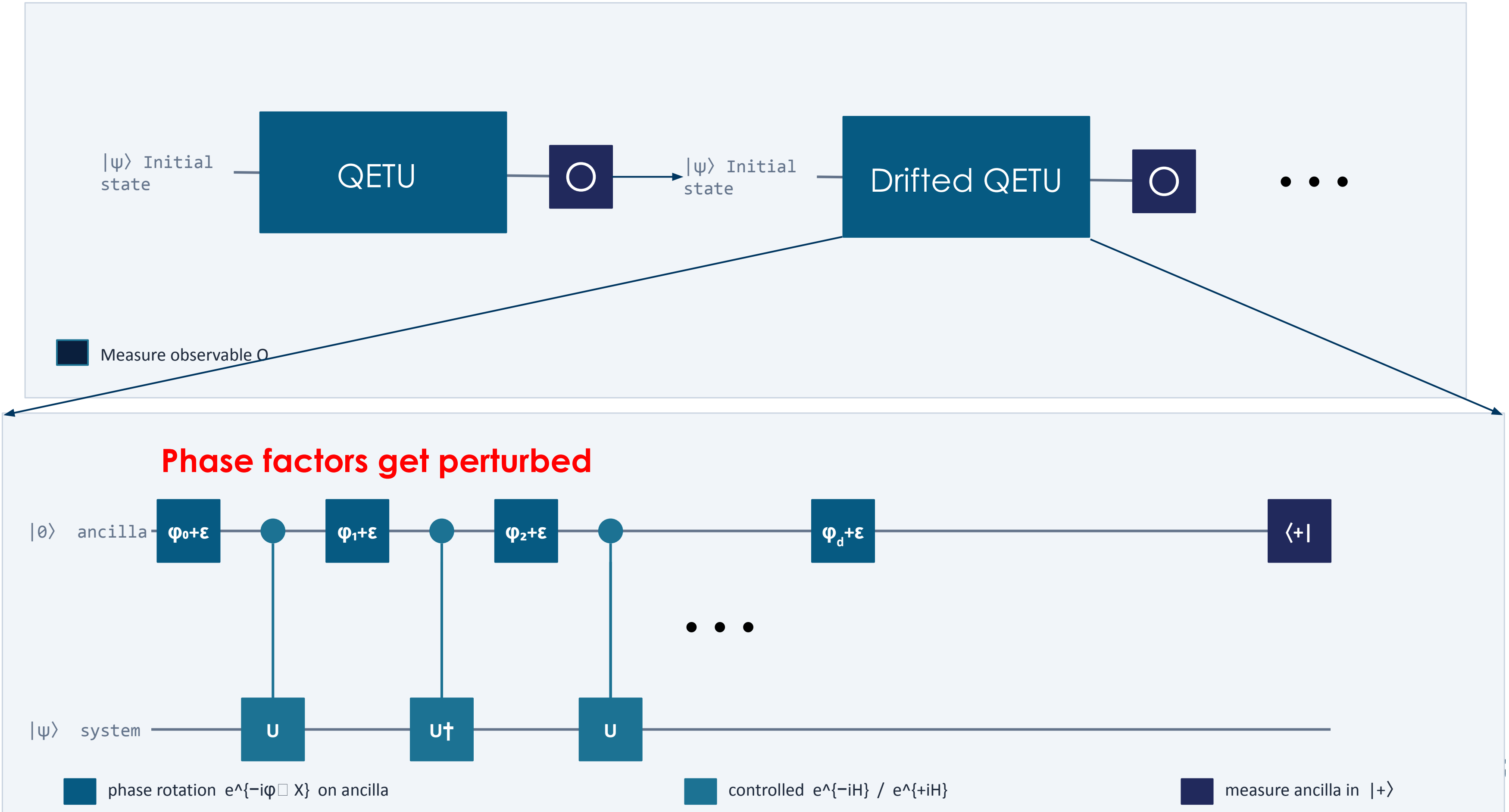
Hardware Defects



Noise Models

Additive

$$\phi^{(\text{noisy})} = \phi_{\text{Wz}} + \sigma \cdot \frac{j}{d_{\text{max}}}$$

Multiplicative

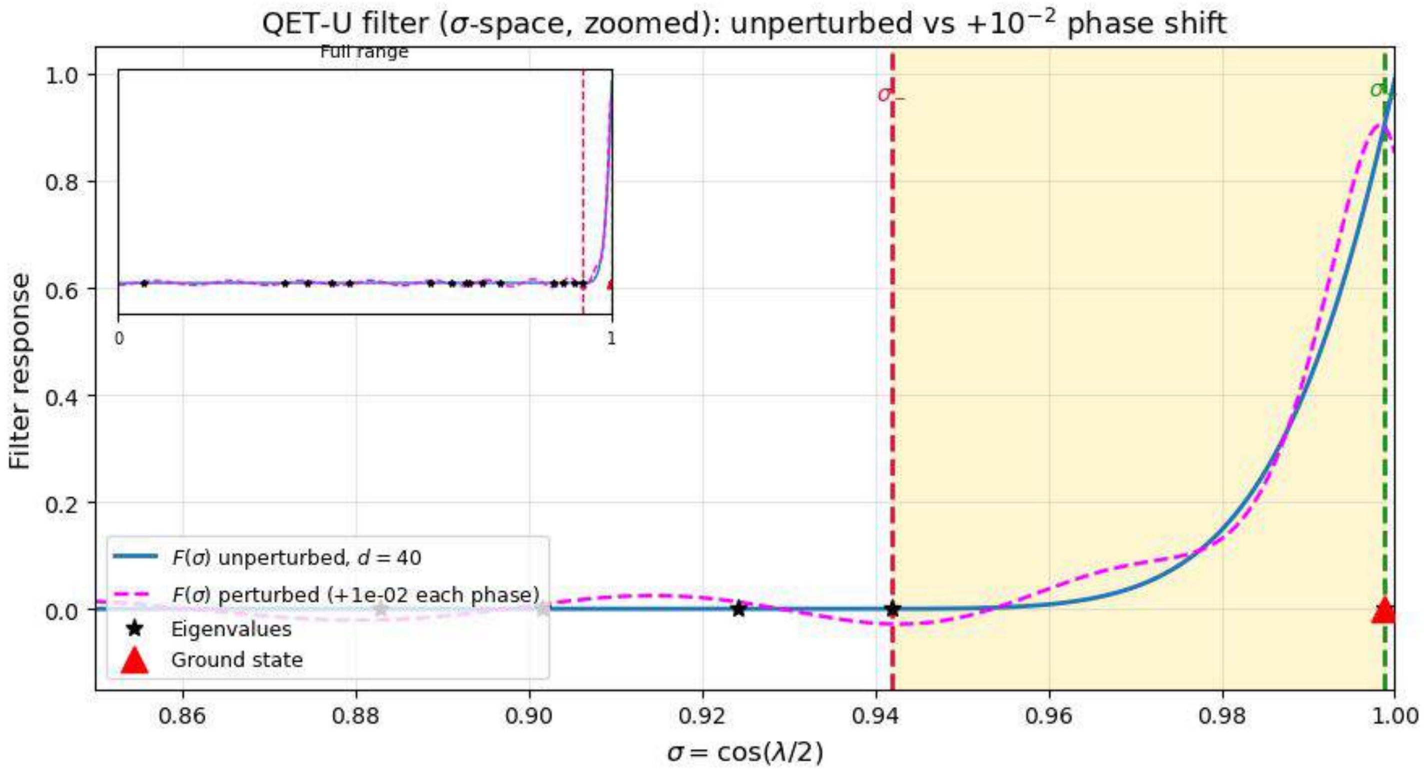
$$\phi^{(\text{noisy})} = \phi_{\text{Wz}} \cdot (1 + \sigma \cdot \frac{j}{d_{\text{max}}})$$

$$\sigma = \text{Drift Rate}$$

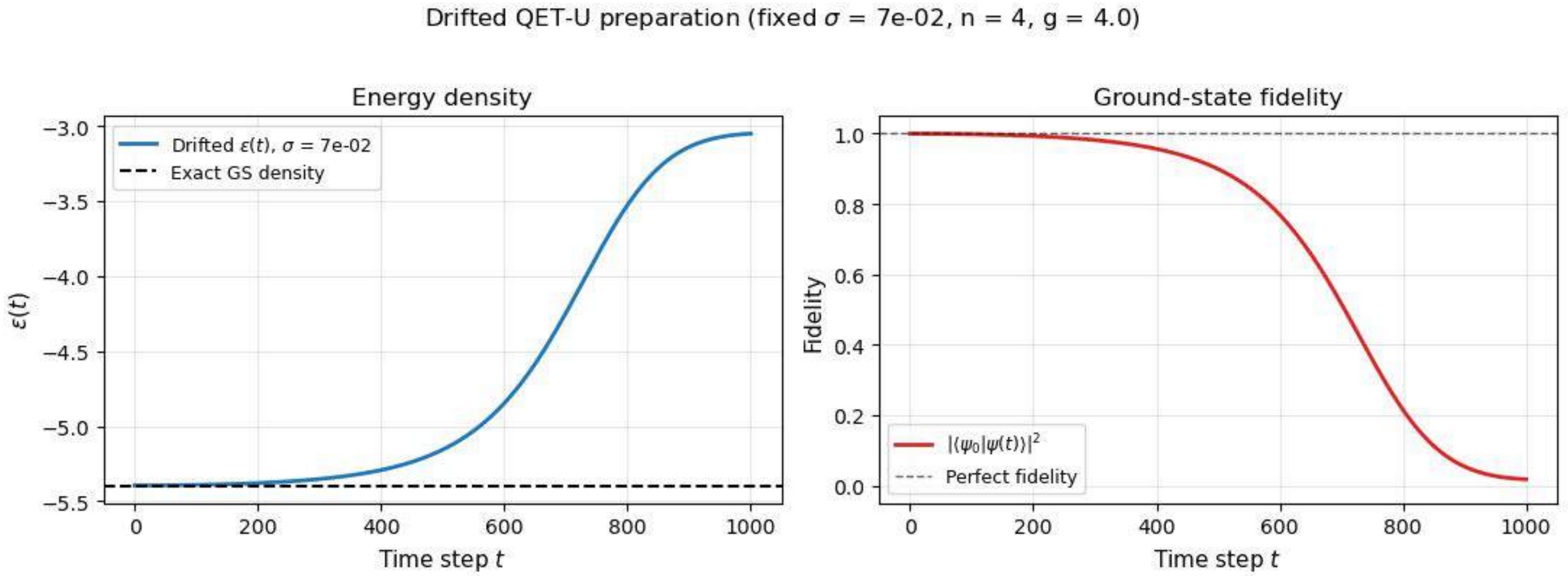
$$j = \text{Current Circuit Iteration}$$

$$d_{max} = \text{Max Circuit Iterations}$$

Noise Models



Noise Models



A Closed Loop Control Scheme

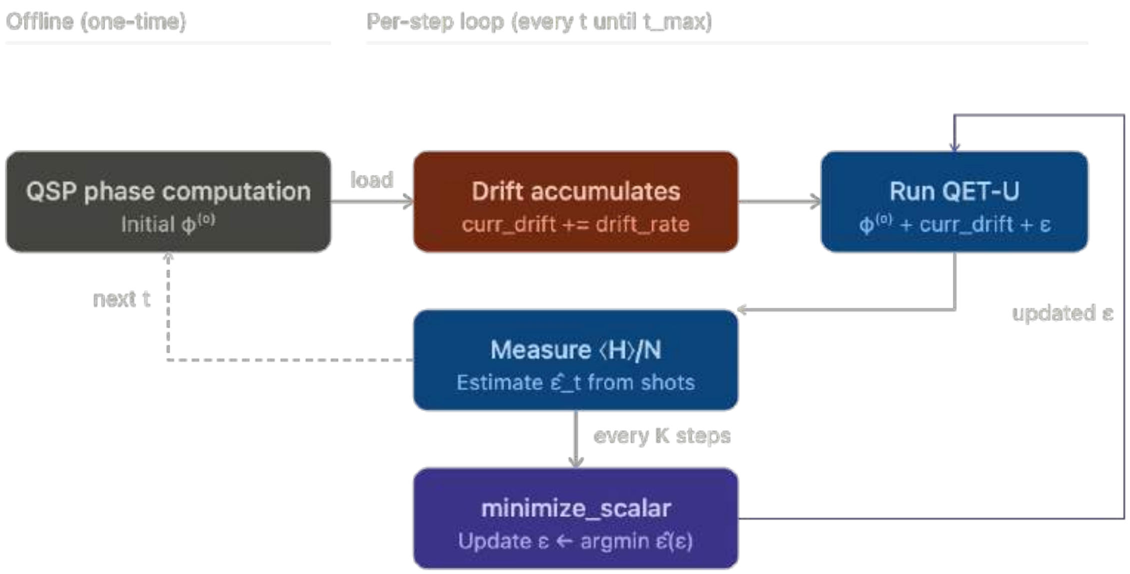
Energy density as a learning signal

- Drift causes the implemented phases $\{\phi'_j\}$ to drift away from the designed $\{\phi_j\}$ → filter degrades → **prepared state has higher energy** than the true ground state.

- This makes $\langle H \rangle / N$ a natural loss function: minimize it by adjusting the phases

- *Closed-loop QET-U: treat phases as learnable parameters, use measured energy density as the objective, optimize online*

Closed loop optimization scheme



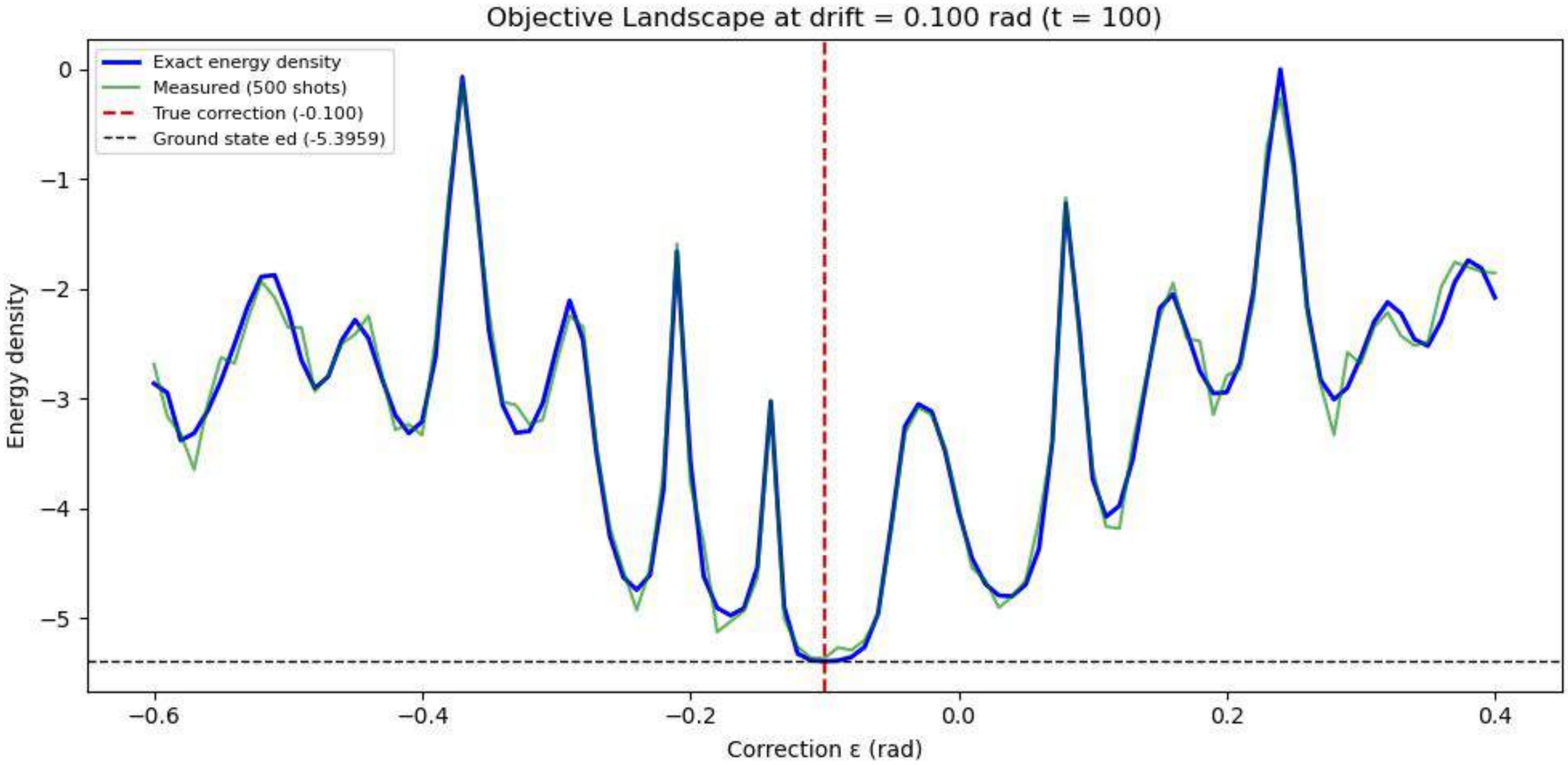
Single-parameter correction

ε is one scalar applied uniformly to every phase — not a per-ϕ_j update

Bounded local search

ε searched in [ε_prev − margin, ε_prev + margin] — avoids oscillatory local minima

Closed loop optimization scheme

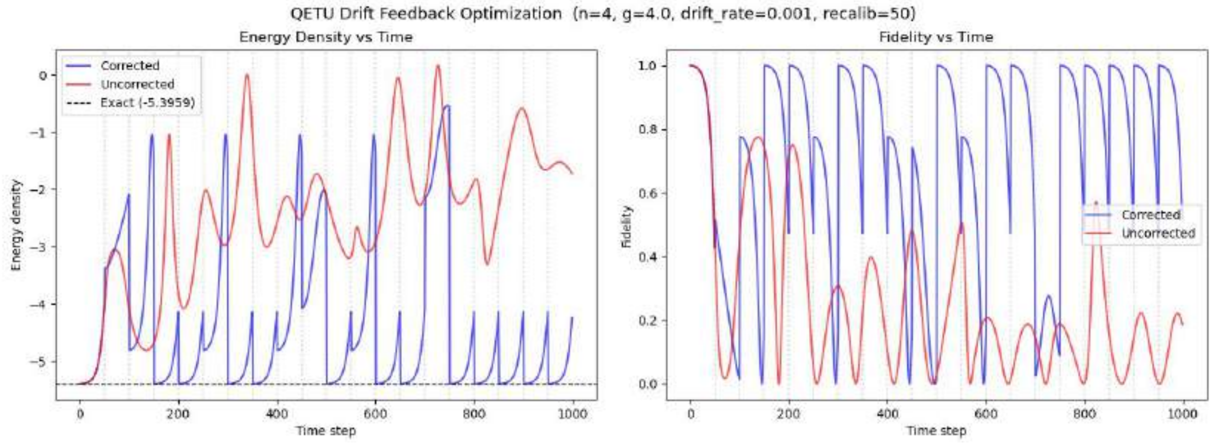


Experimental Setup

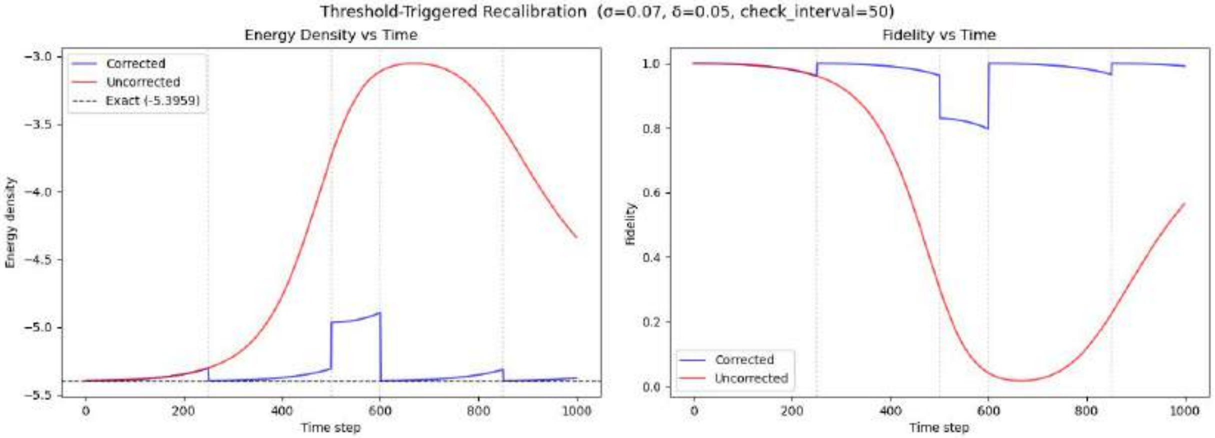
System & Hamiltonian	
TFIM Hamiltonian	$H = -\sum_j Z_j Z_{j+1} - g \sum_j X_j$
System size	$n = 4$ qubits
Transverse field	$g = 4$
Filter degree	$d = 40$
Drift rate	$\delta_{\text{rate}} \in [10^{-4}, 10^{-2}]$ rad/step
Measurement budget	
$\mathrm{N}_{\text{total}} = t_{max} + \left\lfloor \frac{t_{max}}{K} \right\rfloor \cdot E_{\text{opt}} \cdot M_{\text{shots}}$	
Total shots for experiment	N_{total}
Time horizon	t_{max} steps
Recalibration interval	K steps
Optimizer evals per recalibration	E_{opt}
Per-step cost	M_{shots}

Numerical Validation

Additive



Multiplicative



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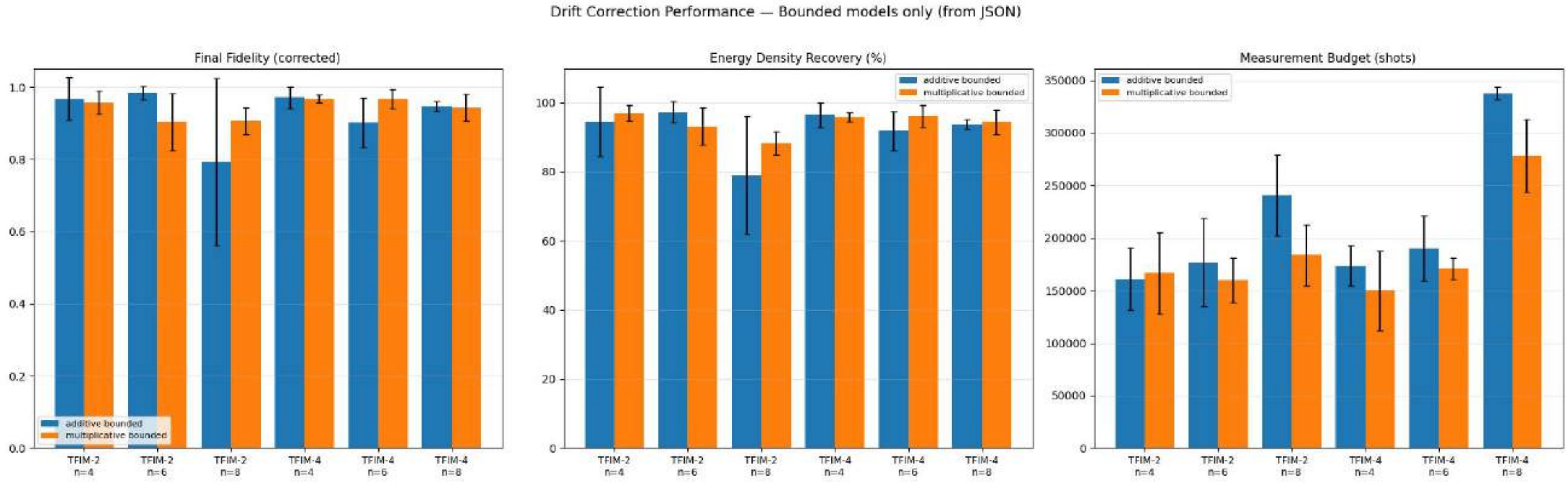
Numerical Validation - scaling



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Future Directions

- Continue Scaling to larger system sizes (20+ qubits)
- Test framework under different hamiltonian (XY model)

References

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